Students' adaptive reasoning in solving geometrical problem

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Students' Adaptive Reasoning in Solving Geometrical Problem

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Abstract. This study aimed to analyze the students' adaptive reasoning ability in solving geometrical problems. This research is classified as a qualitative descriptive study that seeks to describe the analysis of students' mathematical adaptive reasoning. The technique of collecting data is through a mathematical adaptive reasoning ability test and through interviews. The research instrument is the researcher himself and the test questions of adaptive reasoning abilities. The subjects in the study were taken from class VII students because they have studied the material for flat-sided shapes. The subjects selected in this study were S1, S3, and S16. The three subjects are students who work on adaptive reasoning questions and meet the indicators of adaptive reasoning abilities regardless of right and wrong answers. Analysis of qualitative data that is conducted in this study consists of data reduction, data display, conclusion drawing/verification. Based on the analysis of the ability test, S1 can achieve conjecture, provide reasons or evidence for the truth, find patterns from a mathematical problem, check the validity of an argument draw conclusions from a truth. While S3 and S16 can achieve conjecture, provide reasons or evidence for the fact, cannot find patterns from a mathematical problem, and cannot check the validity of an argument, they can conclude a truth. So from the three subjects, it was found that one student fulfilled all the indicators of reasoning ability correctly, while the other two students had not been able to meet two indicators, namely finding patterns from problems and not being able to check the validity of arguments.

INTRODUCTION

Mathematics is a subject that is considered difficult. Solving mathematical problems takes skill, competence, and ability in a students' [1]. The reasoning is one of the logical and systematic thinking skills that are indispensable in learning mathematics. This is in line with which states that five standard processes in mathematics learning consist of problem-solving, reasoning and proof, communication, connection, and representation [2]. So that reasoning becomes one of the abilities that students must possess, therefore mastery of mathematical reasoning accurately and thoroughly is very much needed in learning mathematics in schools, especially adaptive reasoning. In 2001, introduced one reasoning with the term adaptive reasoning [3]. The definition of adaptive reasoning is the ability of students to draw logical conclusions, predict answers, provide explanations about the concepts and procedures of answers used, and assess their truth mathematically [4]. Adaptive reasoning is one of the abilities that must be developed from an early age so that students' intelligence is trained. Adaptive reasoning is not only used by someone in solving problems logically [5]. Inductive and deductive reasoning have been combined by the National Research Council (NRC) in their research in 2001, which was later introduced as adaptive reasoning [6].

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Based on the results of research on the profile of adaptive reasoning, the results are 1) Indicators find patterns, can find patterns in number sequences correctly; 2) Indicators provide reasons for a statement, are unable to give reasons correctly; 3) Indicators making assumptions, unable to understand the meaning of the question so that in compiling the guess the numbers are still wrong; 4) The indicator draws conclusions, is unable to conclude the pattern of the number sequence in accordance with what is asked instead mentions the sequence pattern in Fig. 2 on the question, and; 5) The indicator check validity of the answers, no one did check the validity of answers [7]. According to the National Research Council (NRC), if students have good adaptive reasoning, they will find it easier to learn mathematics, especially in solving mathematical problems [8]. The purpose of adaptive reasoning is to see through various facts, procedures, concepts, and problem-solving methods and to see that everything is correct and makes sense. Therefore, adaptive reasoning is important for students to master or possess to support learning objectives [5][9].

Adaptive reasoning has a wider scope than general reasoning because it includes inductive, deductive, and intuitive reasoning [10]. In achieving higher-order thinking skills, students must be able to develop their adaptive reasoning abilities [11]. This is because adaptive reasoning includes reasoning based on patterns and analogies, logical thinking, and valid evidence in the learning process[10]. In addition, adaptive reasoning refers to the capacity to think logically about the relationship between concepts and situations and ultimately justify by proving the truth of mathematical statements or procedures [4][12].

Suggests that students will be able to develop adaptive reasoning if they meet the following three conditions, namely: 1) New knowledge is inserted after having basic knowledge or adequate prerequisites; 2) The tasks given can be understood and can motivate students, and; 3) The context presented is famous and fun for students. With the fulfillment of these conditions, we expect the students can develop adaptive reasoning. One form of adaptive reasoning is mathematical proof with formal and informal logical reasons. Furthermore, it states that there are five indicators of adaptive reasoning ability, namely: 1) The ability to propose predictions or conjectures; 2) The ability to give reasons for the answers given; 3) The ability to find patterns in a problem; 4) The ability to test the validity of arguments, and; 5) The ability to conclude from statements [4][13].

The current educational process supports rote learning [14], seeking one correct answer without finding other solutions or promoting higher-order thinking [15]. As a result, most students lack adaptive reasoning abilities even though the adaptive reasoning process has been one of the learning objectives since junior high school [16]. Adaptive reasoning is related to a person's ability to process information in visual form, namely visual-spatial intelligence. The purpose of adaptive reasoning is to look through various facts, procedures, concepts, and problem-solving methods and see that everything is correct and makes sense. Therefore, adaptive reasoning is essential for students to master or possess to support learning objectives [5].

METHOD

This research was conducted in June 2021. This research is classified as a qualitative descriptive study that seeks to describe the analysis of students' mathematical adaptive reasoning. The technique of collecting data is through a mathematical adaptive reasoning ability test and through interviews. The subjects in this study were taken from class VII students who had studied the shape of the flat side space. The research instrument is the researcher himself and the test questions of adaptive reasoning abilities. In assessing the validity of the test instruments, we ask for judgments from the experts. The two experts who validated the questions were mathematics education lecturers who had done a lot of research on mathematical thinking ability. According to that in order for the test to have content validity, the following things must be considered: In this case, the experts assess whether the grid made by the test maker has shown that the classification of the grid represents the grid to be measured, namely being able to measure indicators of adaptive reasoning abilities, namely being able to achieve conjecture, being able to provide reasons or evidence for the truth, being able to find patterns from a mathematical problem, being able to check the validity of an argument, being able to conclude a truth. In the next step, the assessors assess whether each test item that has been prepared is suitable or relevant to the grid being tested.

This study uses data analysis in the form of a descriptive narrative using the Miles and Huberman model. Miles and Huberman [17] argue that qualitative data analysis activities are carried out interactively and take place continuously until complete so that the data is saturated. The measure of data saturation is indicated by no longer obtaining new data or information. Qualitative data analysis in this study, namely: 1) Data reduction is the stage of summarizing and focusing the data from the research analysis and eliminating unpatterned data, then the data is collected and selected according to the research objectives; 2) Display data, the reduced data is presented in the form

of a short description so that it is easy to read and understand both in its entirety and parts, and; 3) Conclusion drawing/verification, conclusions are drawn based on the results of the analysis of all the data that has been obtained.

RESULT AND DISCUSSION

This study aims to analyze the adaptive reasoning of class VIII students, from 23 students obtained three students who meet the indicators of adaptive reasoning but regardless of right and wrong answers. The three students who were selected were S1, S3, and S16. The descriptions of the results of the three selected students' adaptive reasoning tests are presented as follows:

The results of the adaptive reasoning ability test for S1 are presented in Fig. 1.

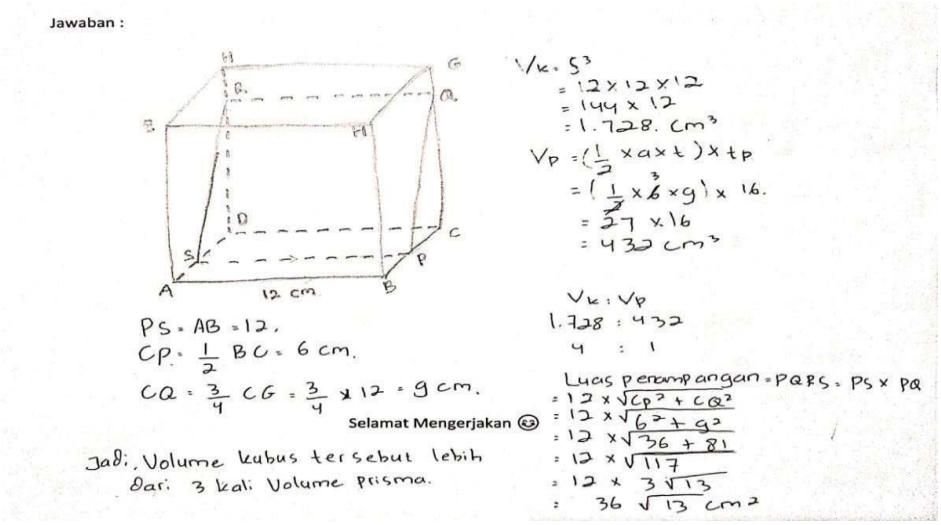

FIGURE 1. The results of the adaptive reasoning for S1.

The adaptive reasoning ability test results for S1 students in solving test questions indicate that they are related to determining assumptions in formulating various possibilities according to their knowledge. This is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*Why do you have to use pictures to solve the problem?*"
S1 : "*To make it easier to do it. Actually, there are several possibilities that the picture doesn't have to be like that. It's just that I took it like that*".

S1 can determine the conjectures that might be resolved in this matter. With S1 describing it, there will be possibilities that can be solved according to his knowledge, when in the interview, S1 explains that the cross-sectional image in the picture can be from various other edges, the determination of the points can also be different. The results of S1 work on adaptive reasoning abilities related to providing reasons or evidence against the truth show that S1 is able to prove the truth. This is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*How do you prove it's true?*"
S1 : "*It can be proven by finding the volume of the cube, then finding the volume of the prism that has been drawn and the length of the side of the cross-section is known, then we can find the other side with Pythagoras because it forms a right triangle, then we can find the volume*."

S1 can prove the reasons or evidence for the truth of the answer by providing calculations proof to find the cube's volume and prism's volume.

The results of S1 work on the adaptive reasoning abilities problem, related to finding patterns from a problem, show that S1 can find patterns from mathematical symptoms to make generalizations is the ability of students to find

patterns or ways of an existing statement. S1 can develop into mathematical sentences. It is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*What is the first step to do?*"
S1 : "*The first one depicts or draws a cube as an illustration*."

Based on the excerpt from the interview above, S1 is able to find the pattern of a mathematical problem, with S1 can find the pattern of the way of a statement so that it can develop into a mathematical sentence by determining the location of the point, describing the cross-section so that it becomes a cube that corresponds to what is known in the text question.

The results of S1 work on adaptive reasoning abilities related to checking the validity of an argument show that S1 can examine an idea, which is an ability that requires students to investigate the truth of an existing statement. It is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*Then how to prove that the volume of the cube is more than three times the volume of the prism?*"
S1 : "*We can see from the results of the calculation of the volume of the cube and prism, then we compare*."

S1 can check the validity of an argument by comparing the cube's volume and prism's volume. It was proven that the volume of the cube was more than three times the volume of the prism. The results of S1 work on adaptive reasoning abilities related to concluding a truth show that S1 can conclude from statements, which is a thinking process that empowers knowledge in such a way as to produce a thought. It is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*So what is the conclusion according to S1*?"
S1 : "*So to find the cross-sectional area, we can see from the picture because it is known that the points P and Q form a line and have a size, so the cross-sectional area is* $36\sqrt{1}cm^2$ *. And the cube's volume is more than three times the volume of the prism as evidenced by the comparison of the two volumes*".

Based on the excerpt from the interview above, S1 can conclude the truth by finding the cross-sectional area and finding the results.

The results of the adaptive reasoning ability test for S3 are presented in Fig. 2.

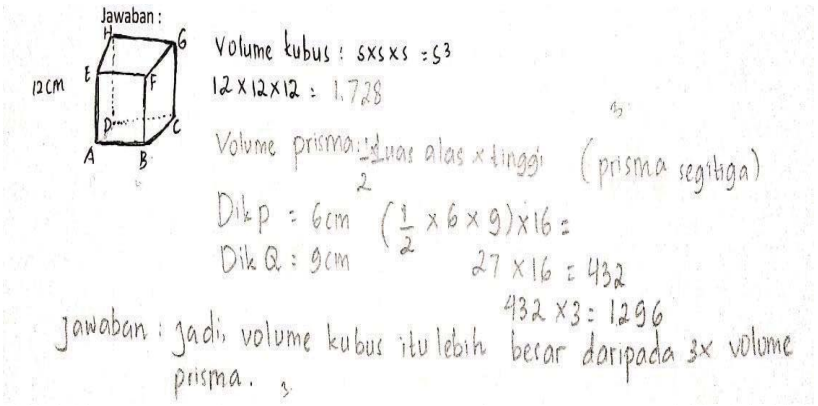

FIGURE 2. The results of the adaptive reasoning for S3.

The results of S3 work on the problem of adaptive reasoning abilities related to determining conjectures show that S1 cannot determine guesses in formulating various possibilities. It is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*Why do you have to use pictures to solve the problem*?"
S3 : "*Because the question is also known as a cube. So it is easier to work on the picture first*."

S3 is not able to determine the assumptions that might be resolved in this matter. With S3 describing it but only with 1 picture, it can't describe another picture, then S3 is less able to determine the guess.
The results of the S3 work on the problem of adaptive reasoning abilities related to providing reasons or evidence against the truth show that S1 can prove the truth. It is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*How do you prove it's true*?"
S3 : "*By calculating the volume of the cube and the volume of the prism*."

Based on the excerpt from the interview above, S3 can prove the reasons or evidence for the truth of the answer by providing proof of calculations to find the cube's and prism's volume. The S3 work result on the Adaptive Reasoning ability problem, related to finding patterns from a problem, shows that S3 cannot find patterns from mathematical symptoms to make generalizations. This ability is about finding patterns or ways of an existing statement to develop into mathematical sentences. It is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*What is the first step to do*?"
S3 : "*Draw a cube according to what is known in the problem*."

Based on the interview excerpt above, S3 is less able to find the pattern of a mathematical problem, with S3 unable to find a pattern or other way of a statement and can only describe one point so that in developing it into a mathematical sentence, it is not perfect.

The results of S3 work on the question of adaptive reasoning ability related to checking the validity of an argument show that a doctorate is less able to examine an argument. This is evidenced by interviews related to researchers on the question of adaptive reasoning ability.

Q : "*Then how to prove that the volume of the cube is more than 3 times the volume of the prism?"*
S3 : "*Judging from the volume of the two wakes."*

Based on the excerpt from the interview above, S3 cannot check the validity of an argument, cannot prove other answers, and is not clear in explaining the results of his calculations.
The results of S3 work on Adaptive Reasoning abilities related to concluding a truth show that S3 can conclude from statements is a thinking process that empowers knowledge in such a way as to produce a thought. It is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*So what is the conclusion according to S3*?"
S3 : "*So to do this problem, we have first to describe the shape and then calculate the volume, and after knowing the two shapes, it is known that the volume of the cube is more than three times the volume of the prism*."

Based on the interview excerpt above, S3 can conclude the truth by looking for the volumes of the two shapes, namely cubes and prisms, and then comparing them to find the results. Analysis of students' adaptive reasoning abilities in terms of checking the validity of an argument. The results of the adaptive reasoning ability test for S16 are presented in Fig. 3.

The results of S16's work on the problem of adaptive reasoning abilities related to determining conjectures show that S16 cannot formulate various possibilities according to their knowledge. It is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*Why do you have to use pictures to solve the problem*?"
S16 : "*Because I think it is necessary to use pictures so that you can imagine what the answer should be*."

Based on the excerpt from the interview above, S16 cannot determine the conjectures that might be resolved in this matter. With S16 describing it but only with one image, there is no other image and only able to determine the points on that cross-section, then S16 is less able to determine guesses.

The results of S16 work on the problem of adaptive reasoning abilities related to providing reasons or evidence against the truth show that S16 can prove the truth. It is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*How do you prove it's true?"*
S16 : "*We just need to find the volume of the known."*

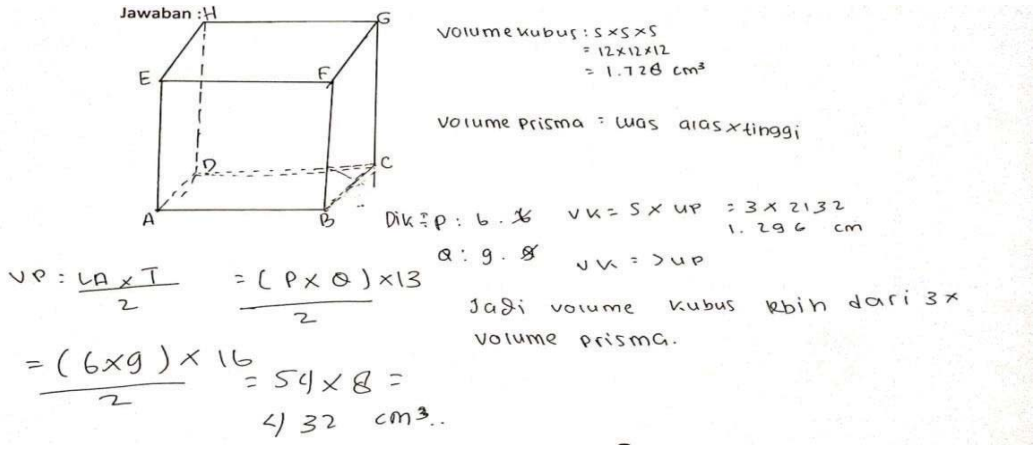

FIGURE 3. The results of the adaptive reasoning for S16.

Based on the excerpt from the interview above, S16 can prove the reasons or evidence for the truth of the answer by providing proof of calculations to find the cube's volume and the prism's volume. The results of S16 work on the problem of Adaptive Reasoning ability related to finding patterns from a problem show that S16 is less able to find patterns from mathematical symptoms to make generalizations. . This is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*What is the first step to do?"*
S16 : "*Draw from what is known in the problem."*

Based on the excerpt from the interview above, S16 is less able to find the pattern of a mathematical problem, S16 cannot find the pattern or method of a statement so that in developing it into a mathematical sentence, it is not perfect. The results of S16's work on the problem of adaptive reasoning ability related to checking the validity of an argument show that S16 cannot check an argument for the truth of an existing statement. It is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*Then how to prove that the volume of the cube is more than three times the volume of the prism*?"
S16 : "*By multiplying the volume of the prism three times*."

Based on the excerpt from the interview above, S16 could not check the validity of an argument by comparing the cube's volume and prism's volume. The results of S16 work on the problem of adaptive reasoning abilities related to concluding a truth show that S16 can conclude from statements is a thinking process that empowers knowledge in such a way as to produce a thought. This is evidenced by interviews related to researchers on the matter of adaptive reasoning abilities.

Q : "*So what is the conclusion according to S16*?"
S16 : "*In conclusion, the volume of the cube is more than three times the volume of the prism*."

Based on the excerpt from the interview above, S16 drew a brief conclusion from the truth by looking for the volumes of the two shapes, namely cubes and prisms, and then comparing them to find the results.

S1's ability to determine conjectures in formulating various possibilities is seen when S1 describes a cube with a cross-section in it. We ensure that S1 can determine the conjecture by interviewing him about the answers to the test

questions that S1 is doing. 1 can provide evidence of the truth of a statement; It can be seen when S1 calculates the cube's and prism's volumes formed in the cube. S1 also finds a pattern or method from an existing statement to develop it into a mathematical sentence; it can be seen when S1 describes the cube, cross-section and determines the points of the cube and the cross-section. Then, we get that S1 can check the validity of the argument by proving it through comparisons, namely, comparing the volume of the cube to the volume of the prism. The last that we get is, S1 which can conclude the statement to get the answer.

Based on the analysis results, S3 is less able to determine allegations in formulating various possibilities. From the S1's answer, we can see that S3 1) Can only show one image without a cross-section; 2) Can provide evidence of the truth of a statement; it can be seen when S3 calculates the calculation of the volume of the cube and prism formed in the cube; 3) Can not determine the pattern or method of an existing statement; it can be seen from the image that is made incomplete; 4) Can not check the validity of the argument by not proving it; S3 is only able to answer without knowing where the results were obtained, and; 5) Can not conclude the statement because it's not finished.

Based on the analysis results, we can see that S16 is 1) Less able to determine conjectures in formulating various possibilities because S16 can only show one image without a cross-section; 2) Able to provide evidence of the truth of a statement; it can be seen when S16 calculates the calculation of the cube's and prism's volumes that are formed in the cube; 3) Less able to determine the pattern or method of an existing statement; it can be seen from the image that is made incomplete; 4) Less able to check the validity of the argument by not proving it; 5) Only able to answer without knowing where the results were obtained and; 6) Able to conclude briefly from the truth by looking for the volume of the two shapes, namely, the cube and the prism, and then compared to find the result. It is in line with Oktaviana and Haryadi [18]. They concluded that students' adaptive reasoning abilities were still in the low to the very low category because 25 students (69.45%) were only able to be in the low category with a score range of 8-16 (from a maximum score of 26). Therefore, the student's adaptive reasoning really needs to be improved so that the expected learning objectives can be achieved and make mathematics learning more meaningful.

CONCLUSION

Based on the test analysis result, S1 can achieve conjecture, provide reasons or evidence of truth, find patterns of a mathematical problem, check the validity of an argument, and draw conclusions from a fact. In comparison, S3 and S16 can achieve being able to formulate conjectures, provide reasons or evidence for the truth, and find patterns of a mathematical problem. But they can not check the validity of an argument able to conclude a truth.

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